Performance Based Learning and Assessment Task

Connecting theoretical and experimental probability

I. ASSESSMENT TASK OVERVIEW & PURPOSE:

In this task students will explore theoretical and experimental probability and derive the law of large numbers. Students will be asked to determine the theoretical probability of an event of their choosing (ie: rolling a dice, flipping a coin, drawing a card). They will calculate the theoretical probability of what they consider a success. Then they will make predictions of how many successes they think will occur for a given number of experiments, perform the experiments, and calculate experimental probability. After that they will draw conclusions about the connection between theoretical and experimental probability. Next the class data from all the groups will be combined and students will discuss if the law of large numbers was evident in the combined class data. Lastly, they will extend their conclusions and research real-world applications of the law of large numbers.

II. UNIT AUTHOR:

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III. COURSE:

Algebra, Functions, and Data Analysis

IV. CONTENT STRAND:

Probability

V. OBJECTIVES:

The student will be able to:

- Determine theoretical probability
- Make predictions and perform experiments and then calculate experimental probability
- Draw conclusions about the connection between theoretical and experimental probability
- Extend conclusions and research real-world applications of the law of large numbers

VI. REFERENCE/RESOURCE MATERIALS:

Multiple standard decks of playing cards
Multiple sets of more than 25 dice
Multiple coins
Large presentation paper
Classroom set of calculators

VII. PRIMARY ASSESSMENT STRATEGIES:

The task includes an assessment component that performs two functions: (1) for the student it will be a checklist and provide a self-assessment and (2) for the teacher it will be used as a rubric. The assessment list is designed to also act as a rubric which the student can use as a checklist and then self-assess. This assessment list will be used to assess the student's work.

VIII. EVALUATION CRITERIA:
Rubrics and a self-assessment scoring sheet are included at the end. Also attached is a benchmark of exemplary work.

IX. INSTRUCTIONAL TIME:
This activity should take approximately three 90-minute blocks.

Probability

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Probability

Mathematical Objective
The mathematical goal of this activity is for the students to realize the law of large numbers on their own. They will explore theoretical and experimental probability and extend their conclusions to create the law themselves. Then they will explore real-world examples of the law of large numbers. Students are expected to conclude that the law of large numbers is used for life expectancy, health insurance, gambling, etc.

Related SOL
- AFDA.6 – The student will calculate probabilities. Key concepts include (e) the Law of Large Numbers.

NCTM Standards
The student will:
- Understand and apply basic concepts of probability
- Use a basic understanding of probability to make and test conjectures about the results of experiments and simulations.
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others

Materials/Resources
Multiple standard decks of playing cards
Multiple sets of more than 25 dice
Multiple coins
Large presentation paper
Classroom set of calculators

Assumption of Prior Knowledge
- Students should have successfully completed Algebra 1 before taking this class.
- Students should know the difference between experimental and theoretical probability but have not been exposed to the law of large numbers.
- Students should be operating on the Informal deduction level on the Van Hiele scale with respect to probability.
- Students should have prior mathematical knowledge that includes working with fractions and ratios.

Day 1 – Making Predictions

Introduction: Setting Up the Mathematical Task

In this activity, you will investigate the relationship between theoretical and experimental probability. You all know the difference between theoretical and experimental probability. Sometimes they're equal, sometimes they're not. Now we will conduct some experiments to see if there is a relationship between them. Think about something you can make a prediction about. Some examples are flipping a coin, drawing a card from a standard deck, rolling a dice, etc. Get into groups of two or three based on what you chose. Groups should not exceed three and it is ok if there are multiple groups exploring the same experiment. (In other words, it is better to have multiple groups exploring the same experiment rather than have groups exceeding three members.)

Student Exploration

Small Group Work

1. In groups students will discuss and decide upon their predicted outcome, in other words, what they consider a success and record it on their worksheet.
2. The students will calculate the theoretical probability of their event occurring.
3. Each group will perform their experiment one time and record the result.
4. The teacher will instruct the students to complete the table on the worksheet. It includes students making predictions before repeating their experiment a certain number of times and then recording the results of what actually happened.
5. The students will explain why they made the predictions they did.
6. In preparation to perform the experiment multiple times, the students will predict what they think their percentage of success outcomes will be.
7. The teacher will set a timer for 30 minutes and instruct the students to perform their experiment as many times as possible. Students must keep track of the outcomes.
8. Together, each group will write a paragraph about conclusions they can draw, if there is a general rule. If so, will that general rule extend to other experiments?
9. When all the groups have finished filling out their worksheets, they will prepare to present their findings to their classmates by writing their results on the large poster paper (one per group).

The teacher should tell the students to write a law or rule that will be shared with the class the next day. They should make their law as specific as possible and use appropriate mathematical terminology.

If students get stuck with this, the teacher should prompt them by suggesting they include the terms experimental probability and theoretical probability. The goal here is to get them to discover the law of large numbers, which states that the more times an experiment is conducted, the closer the experimental probability will get to the theoretical probability.

Student/Teacher Actions:

- Students should be filling out their worksheets while in their groups.
- The teacher should be walking around the classroom checking on the groups to make sure everyone is on task.

Day 2 – Explorations

Introduction: Setting Up the Mathematical Task

Today we will be sharing and then testing our results from yesterday's experiments. Sit at a table with your group from yesterday. First, we will present our results from yesterday.

(After that is complete continue) At each table there are either some decks of cards, dice or fair coins. You will conduct a different experiment from what you did yesterday. Then we will add up all the results from all the groups and compile the information into one huge probability to see if our class can make the law of large numbers work.

Student Exploration

Small Group Work

- Groups will take turns presenting their findings from the experiments.
- Each group should decide what a "success" is and then run the experiment as many times as possible within a 30 minute time frame.

Student/Teacher Actions:

After the groups have completed the experiments, the teacher should facilitate the compiling of a class list of outcomes.

Dice	
# of successes	# of flips

Coins	
# of successes	# of tosses

Cards	
# of successes	# of drawings

Have the students (still in their groups) recall the theoretical probability for each event.

- Dice $\frac{1}{6}$
- Coins $\frac{1}{2}$
- Cards $\frac{4}{52}$ or $\frac{1}{13}$

Tell them to discuss within their groups what they observe about the combined class data. (that it illustrates the law of large numbers) Prompt them to recall the "law" they wrote down the day before.

Day 3 – Real World Extensions

Introduction: Setting Up the Mathematical Task

- Students will sit in the same groups as yesterday.
- They will finish writing on their poster paper if they did not finish the day before.
- The teacher will decide the order in which the groups will present their findings.

Student Exploration

Small Group Work

- After presentations are concluded, students will begin to research (still in their groups) real-world applications of the law of large numbers.
- Students must find an industry and be able to clearly explain a specific example of how the law is used in that industry.
    - The expected responses are in the industries of life-insurance premiums, healthcare, and gambling.
- After each group finds a real-world example, the groups will again take turns telling the rest of the class about their findings.

Student/Teacher Actions:

- Students should participate in the presentation of their experiments
- Students should participate in researching real-world applications for the law of large numbers.
- Students should participate in the presentation of their real-world findings to the rest of the class.
- The teacher should walk around the classroom checking on the groups to make sure everyone is on task.

Monitoring Student Responses

- Students will communicate verbally their findings and share with the class. They will share the results from their experiment and the real-world applications they discovered.

Benchmarks

- At the end of Day 1 the students will explain the law of large numbers in their own words.
- At the end of Day 2 the class will combine the data from their experiments and explore whether the law of large numbers worked.
- At the end of Day 3 the class will end with the presentation of real-world examples of the law of large numbers.

Assessment List

- Students will be assessed on completion of their worksheet while conducting their experiment
- Students will be assessed on their presentation of findings to their classmates
- Students will be assessed on their discovery of real world examples of the law of large numbers and on their presentation of that to the class.

Day 1 - Making Predictions Worksheet

Group members: _____

Event (ie: drawing a card, rolling a dice, flipping a coin): _____

What is considered a "success": _____

1. State the theoretical probability of your event.

2. Perform the experiment one time. Using complete sentences, describe how your result compares to the theoretical probability.

3. Complete the table below.

	Predicted number of success before experiment	number of times experiment conducted	results (number of successes)	Experimental Probability
a.		3		
b.		11		
c.		25		

4. Write a detailed explanation of why you predicted what you did. Were the predictions for rows b and/or c affected by the outcome in row a?

5. Repeat the process again but you decide how many times you will conduct the experiment (must be more than 25). Make sure you predict before conducting the experiment.

	Predicted percent of successes before experiment	number of times experiment conducted	results (number of successes)	Experimental Probability
d.				

6. Write a paragraph about what conclusions you can draw. Is there a general rule? Will the general rule extend to other types of experiments? Explain.

Rubrics

Rubric for the "Making Predictions" worksheet

Worksheet is complete	0 – The worksheet is incomplete, missing some items.	1 – The worksheet is complete except for one question (or one part of a question)	2 – All questions are complete on the worksheet
Questions 2,4,6 are answered thoughtfully and in complete sentences	0 – Questions 2,4,6 are not answered	1 – Questions 2,4,6 are answered but not in complete sentences or not thoughtfully	2 – Questions 2,4,6 are answered in complete sentences and are well thought out responses.
Probabilities are accurately calculated	0 – Probabilities are not calculated	1 – Some calculations are missing or wrong	2 – All calculations are accurate and complete

Rubric for group presentation of experiment

Poster paper is neat and organized	0 – Poster paper is incomplete	1 – Poster paper is not neat or not well organized	2 – Poster paper is neat and well organized, and is easy to follow.
All group members contributed in presenting	0 – 2 or more members did not contribute to the presentation	1 – only one member did not contribute to the presentation	2 – all group members contributed to the presentation
Group members were on task	0 – group members needed multiple reminders to stay on task	1 – group members needed one reminder to remain on task	2 – group members worked without teacher intervention

Rubric for real world examples and presentation

All group members worked to find real-world examples	0 – 2 or more members did not work on finding examples	1 – only one member did not work on finding examples	2 – all group members worked diligently on finding real world examples
Group presentation of example to class	0 – 2 or more members did not contribute to the presentation	1 – only one member did not contribute to the presentation	2 – all group members contributed to the presentation
Ability to answer questions asked by teacher or peers	0 – no group members were able to answer questions asked by the class or teacher	1 – some group members were not able to answer questions asked by the class or teacher	2 – all group members were able and willing to answer questions asked by the class or teacher

Self-Assessment and Teacher Assessment Table

Task		Points possible	Self - Assessment	Teacher Assessment
experiment worksheet	Worksheet is complete	2		
	Questions 2,4,6 are answered thoughtfully and in complete sentences	2		
	Probabilities are accurately calculated	2		
group presentation of experiment	Poster paper is neat and organized	2		
	All group members contributed in presenting	2		
	Group members were on task	2		
real world examples and presentation	All group members worked to find real-world examples	2		
	Group presentation of example to class	2		
	Ability to answer questions	2		
TOTAL		18		

Day 1 - Making Predictions Worksheet

Group members: Example of exemplary student work

Event (ie: drawing a card, rolling a dice, flipping a coin): flipping a fair coin

What is considered a "success": heads

1. State the theoretical probability of your event.

P(heads) = $\frac{1}{2} = .5$ *or* 50%

2. Perform the experiment one time. Using complete sentences, describe how your result compares to the theoretical probability.

Performing the experiment one time means the coin was flipped one time. For this single experiment the coin landed on tails, which is the opposite of what is considered a success. Therefore, the experimental probability of heads was 0.

3. Complete the table below. Each line is a separate experiment, therefore, the three times in row a are not included in the eleven times in row b.

	Predicted number of success before experiment	number of times experiment conducted	results (number of successes)	Experimental Probability
a.	2	3	1	$\frac{1}{3} = .33$ *or* 33%
b.	6	11	5	$\frac{5}{11} = .45$ *or* 45%
c.	14	25	13	$\frac{13}{25} = .52$ *or* 52%

4. Write a detailed explanation of why you predicted what you did. Were the predictions for rows b and/or c affected by the outcome in row a?

We predicted as close as possible to half of the number of times we were going to conduct the experiment. The reason we were predicting half is because the theoretical probability of flipping heads is 50%. No, the predictions for rows b and c were not affected by the outcome in row a. We thought the probability would be 50% every time.

5. Repeat the process again but you decide how many times you will conduct the experiment (must be more than 25). Make sure you predict before conducting the experiment.

	Predicted percent of successes before experiment	number of times experiment conducted	results (number of successes)	Experimental Probability
d.	15	30	16	$\frac{16}{30} = .53$ *or* 53%

6. Write a paragraph about what conclusions you can draw. Is there a general rule? Will the general rule extend to other types of experiments? Explain.

The more times we conduct the experiment, the closer the experimental probability gets to the theoretical probability. It seems obvious that the rule will extend to other types of experiments because there is nothing unique about flipping a coin versus rolling a dice or drawing a card. All of those scenarios have a theoretical probability and an experimental probability after an experiment is conducted.

Day 3 – Real World Extensions

Benchmark – example of exemplary student work (on poster paper)

REAL WORLD APPLICATION OF THE LAW OF LARGE NUMBERS

The law of large numbers is used by Blaine Insurance to set appropriate rates on auto insurance.

1. The company studies a large number of people in a certain population, for example, 18 year old males.
2. They use their observations to make predictions about the likelihood a member of that population will get into an accident.
3. Those predictions (along with other factors) are then used to set rates so they charge more for higher-risk drivers.

Source: http://blaineinsurance.com/blog/insurance-rates-and-the-law-of-large-numbers